AMS210.01.
Homework 1

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Due at the beginning of the class, February 17, 2003

1. Solve each of the the following equations:
   (a) $5x = \log_3 2$
   (b) $0x = 3$
   (c) $0x = 0$
2. Solve each of the following equations. Consider a as a parameter and don't forget different cases for different values of it:
   (a) $6x = a$
   (b) $ax = 2$
   (c) $ax = 2a$
3. Write the solution set of each of the following equations and find 2 particular solutions:
   (a) $x_1 + x_2 = 6$
   (b) $2x_1 - x_2 + 4x_3 = -1$
4. Solve the following systems in echelon form. Also, for each system which has infinitely many solutions, specify 1 numerical solution.

   (a) $\begin{cases} x_1 + 3x_2 - x_3 = 13 \\ x_2 + 3x_3 = 10 \\ -2x_3 = -4 \end{cases}$

   (b) $\begin{cases} x_1 - 4x_2 + 10x_3 = 3 \\ 15x_2 + 3x_3 = 10 \\ 0 = 2 \end{cases}$

   (c) $\begin{cases} x_1 + 4x_2 + x_3 = 15 \\ -3x_3 = -3 \\ 0 = 0 \end{cases}$

(d) $\begin{cases} 2x_1 - x_2 + x_3 = 15 \\ x_2 + 2x_3 + 2x_4 = 10 \\ 2x_4 = 6 \end{cases}$

(e) $\begin{cases} 2x_1 + 2x_2 + x_3 - x_4 = 4 \\ x_3 + 3x_4 = 8 \end{cases}$

5. Find the elementary operation needed to get from the first system to the second one:

(a) $\begin{cases} x_1 + 2x_2 + x_3 = 9 \\ 2x_1 - x_2 + 3x_3 = 5 \\ 3x_1 + x_2 - x_3 = 4 \end{cases} \Rightarrow \begin{cases} x_1 + 2x_2 + x_3 = 9 \\ 3x_1 + x_2 - x_3 = 4 \\ 2x_1 - x_2 + 3x_3 = 5 \end{cases}$

(b) $\begin{cases} x_1 + 2x_2 + x_3 = 9 \\ 2x_1 - x_2 + 3x_3 = 5 \\ 3x_1 + x_2 - x_3 = 4 \end{cases} \Rightarrow \begin{cases} x_1 + 2x_2 + x_3 = 9 \\ 2x_1 - x_2 + 3x_3 = 5 \\ 5x_1 + 2x_3 = 9 \end{cases}$

(c) $\begin{cases} x_1 + 2x_2 + x_3 = 9 \\ 2x_1 - x_2 + 3x_3 = 5 \\ 3x_1 + x_2 - x_3 = 4 \end{cases} \Rightarrow \begin{cases} x_1 + 2x_2 + x_3 = 9 \\ 6x_1 - 3x_2 + 9x_3 = 15 \\ 3x_1 + x_2 - x_3 = 4 \end{cases}$

(d) $\begin{cases} x_1 + 2x_2 + x_3 = 9 \\ 2x_1 - x_2 + 3x_3 = 5 \\ 3x_1 + x_2 - x_3 = 4 \end{cases} \Rightarrow \begin{cases} x_1 + 2x_2 + x_3 = 9 \\ 2x_1 - x_2 + 3x_3 = 5 \\ 2.5x_2 - 5.5x_3 = -3.5 \end{cases}$

(e) $\begin{cases} x_1 + 2x_2 + x_3 = 9 \\ 2x_1 - x_2 + 3x_3 = 5 \\ 3x_1 + x_2 - x_3 = 4 \end{cases} \Rightarrow \begin{cases} x_1 + 2x_2 + x_3 = 9 \\ - 5x_2 + x_3 = -13 \\ 3x_1 + x_2 - x_3 = 4 \end{cases}$

6. Solve each of the following systems. Also, for each system which has infinitely many solutions, specify 1 numerical solution.

(a) $\begin{cases} x_1 - 3x_2 = \frac{1}{2} \\ 2x_1 + 2x_2 = 13 \end{cases}$

(b) $\begin{cases} x_1 + 2x_2 + x_3 = 9 \\ 2x_1 - x_2 + 3x_3 = 5 \\ 3x_1 + x_2 - x_3 = 4 \end{cases}$

(c) $\begin{cases} 2x_1 + 2x_2 + 2x_3 = 10 \\ x_1 + 3x_3 = 5 \end{cases}$

(d) $\begin{cases} 2x_1 + 2x_2 + 2x_3 = 10 \\ 2x_1 + 4x_2 + 3x_3 = 7 \\ 6x_1 + 10x_2 + 8x_3 = 25 \end{cases}$

(e) $\begin{cases} 8x_1 + x_2 + x_3 = 1 \\ x_1 + 8x_2 + x_3 = 1 \\ x_1 + x_2 + 8x_3 = 1 \end{cases}$

(f) $\begin{cases} 12x_1 + 9x_2 + 3x_3 + 10x_4 = 13 \\ 4x_1 + 3x_2 + x_3 + 2x_4 = 3 \\ 8x_1 + 6x_2 + 2x_3 + 5x_4 = 7 \end{cases}$

(g) $\begin{cases} 2x_1 + 5x_2 - 8x_3 = 8 \\ 4x_1 + 3x_2 - 9x_3 = 9 \\ 2x_1 + 3x_2 - 5x_3 = 7 \\ x_1 + 8x_2 - 7x_3 = 12 \end{cases}$

7. Find the reduced row echelon form of the following systems:

(a) $\begin{cases} 2x_1 + 2x_2 + 2x_3 + 6x_4 = 5 \\ x_3 + 2x_4 = -1 \end{cases}$

(b) $\begin{cases} -2x_1 - 2x_2 + x_3 + x_4 = 5 \\ x_3 + x_4 + x_5 = 0 \\ - x_4 - 2x_5 = -1 \end{cases}$

8. The system

$$\begin{cases} -2x_1 + x_2 + 3x_3 + x_4 = 2 \\ 2x_2 + x_3 - 4x_4 + 3x_5 = 0 \\ 3x_1 - x_2 - x_4 - 2x_5 = 0 \\ x_1 + 4x_2 - 7x_3 + 3x_4 = -3 \\ 4x_1 - x_3 - 4x_4 = -1 \end{cases}$$

has unique solution $(x_1, x_2, x_3, x_4, x_5) = (1, 0, 1, 1, 1)$. Determine the reduced row echelon form of this system.

9. Write a system with the following solution set:

$$\{ (4 + 2k_2; k_2; 5), k_2 \in \mathbb{R} \}$$

10. A **monic polynomial of degree 2** is a function $f(x) = x^2 + px + q$. Find monic polynomials of degree 2 such that:

(a) $f(1) = 6$

(b) $f(0) = 1; \quad f(2) = 5$

In each part explain is the found polynomial unique or not.

11. A **polynomial of degree 2** is a function $f(x) = ax^2 + bx + c$. Find polynomial of degree 2 such that: $f(0) = 1; \quad f(1) = 4; \quad f(-1) = 2$. Explain is this polynomial unique or not.

12. [**Extra credit**] For each value of λ solve the following system. Don't forget to consider different cases!

$$\begin{cases} \lambda x_1 + x_2 + x_3 = 1 \\ x_1 + \lambda x_2 + x_3 = 1 \\ x_1 + x_2 + \lambda x_3 = 1 \end{cases}$$